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NP-13186

CF-2954
May 17, 1962
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NUMERICAL APPROXIMATIONS IN HEAT TRANSFER PROBLEMS AND USAGE OF IBM 7090 COMPUTER FOR SOLUTIONS

by

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(5) 69 100

(14) Repl. no. CF-2954

(6) Numerical Approximations in Heat Transfer Problems and Usage of IBM 7090 Computer for Solutions,

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The Johns Hopkins University
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Numerical Approximations in Heat Transfer Problems and Usage of IBM 7090 Computer For Solutions

I. Introduction

The purpose of this paper is to furnish information necessary to obtain numerical approximations to the solutions of certain heat transfer problems by making use of a thermal model and related IBM 7090 computer program developed by BBE and BCC. The solutions take the form of a time history of temperature distribution. The problems concern heated structures or components of quite general geometric configuration and material composition.

Material properties may be temperature dependent. Heating may occur by convection, by radiation, or by conduction. In the case of aerodynamic heating, provision is made for a real gas boundary layer with automatic determination of whether the flow is laminar or turbulent. The trajectory and ambient atmosphere may be arbitrarily specified.

In section II, we consider a thermal model that consists of a lumped parameter network of thermal capacitances and conductances. This model may be thought of as arising from the replacement of the differential equations of heat conduction and their boundary conditions by a suitable set of difference equations or from the replacement of a continuous heat conduction system by a corresponding lumped parameter system on the basis of physical considerations. Our goal here is not to discuss the limitations of the lumped parameter model*, but rather to describe the equations which govern it. In the appendices we give first the details of the computations involved in the calculation of aerodynamic heating. Next, we give a criterion to evaluate the stability of the calculations. Then, we show the difference equations.

In section III, we consider the specification of this model via a very simple but complete FORTRAN control program. The existence of a certain

*For an excellent treatment of the derivation of lumped parameter heat transfer models see Dusenberre's Numerical Analysis of Heat Flow, McGraw-Hill Book Co., Inc., 1949.

set of FORTRAN-FAP subroutines in the BCC Library (Nos. 11.02.01-11.02.12) then makes direct liaison between the BCC Operations Project and the user a relatively simple matter which requires virtually no knowledge of programming on the part of the user. The dimensional units to be used in each case are listed in Appendix D.

Given a specific problem, one proceeds by setting up the model along the lines indicated in section II and specifying the model along the lines indicated in section III. In section IV the printout format is discussed. Finally, a typical example is carried through in detail in section V. The last appendix gives the limitations on the size of the program and the estimates which are to be used in computing the machine time.

II. The Thermal Model and Its Equations

The thermal models that we consider consist of networks of thermal capacitances and conductances. In any model, to each end point of a thermal conductance is assigned a number from 1 to 1000 called an index. Each such point is called an indexed point and may have assigned to it a value of thermal capacitance or a temperature prescribed as a function of time. In any case the temperatures which are associated with the indexed point s, for example, are designated T_s. Similarly, the value of a thermal capacitance associated with an indexed point r is designated C_r, and the value of a conductance joining the indexed points r and s is designated by K_{rs}. In general the values of each thermal capacitance, C_s, will be dependent on the temperature, T_s; and the values of any thermal conductance K_{rs} will depend on T_r, T_s, and time. By allowing this generality the thermal capacitances can represent, over an extended temperature range, the thermal capacities of pieces of solids; and the thermal conductances can represent the behavior of the thermal conductivities of solids, of convective heat transfer, and of thermal radiation.

Within the network at any time there will be a thermal flux between each pair of points joined by a thermal conductance. The value of the flux, q_{rs}, from r to s, for example, is given by an apparently linear relation,

$$q_{rs} = K_{rs} (T_r - T_s) . \qquad (1)$$

The linearity of this relation is truly only apparent, since K_{rs} may be temperature dependent. To determine the total thermal flux, q_s, into the point s, it is necessary only to sum all the q's which have s for their second index. This is expressed by writing

$$q_s = \sum_r q_{rs} \tag{2}$$

Further, in a short period of time Δt, the total thermal energy into the point s is $q_s \cdot \Delta t$. Consequently, if s is a point with which a thermal capacitance C_s is associated, then the change in temperature ΔT_s in interval Δt is computed by the relation

$$\Delta T_s = \frac{q_s \, \Delta t}{C_s} \quad . \tag{3}$$

Here C_s is presumed to depend on the value of T_s at the beginning of the time interval, the precise nature of this dependence will be discussed later.

If the temperatures of all indexed points are known at a time t and if the values of the C_s and K_s are also known, then the relations (1), (2), and (3) give the values of the temperatures of all indexed points which have capacitances associated with them at a slightly later time $(t + \Delta t)$. At the remainder of the indexed points the temperatures at $(t + \Delta t)$ are prescribed or calculated directly as functions of time, so that by step by step computations the temperatures can be obtained at all indexed points at later times.

The general description of the thermal network must be completed by prescribing the computation of the values of the conductances and capacitances. Of these elements the simplest in form is the thermal capacitance. For any of these, say C_s, the defining equation is

$$C_s = V_s \cdot (\rho C_p)_s \, , \tag{4}$$

where V_s is a prescribed constant having the dimensions of volume, and $(\rho C_p)_s$ is given either by a polynomial in T_s degree five or less or by interpolating in a table, and has the dimension of thermal capacity per unit volume.

The thermal conductances may be of any of several types. Of these the simplest has the form

$$K_{rs} = \left(\frac{A}{L}\right)_{rs} \cdot k_{rs} , \qquad (5)$$

where A and L are the area and length respectively, and k_{rs} is a thermal conductivity which depends on temperature. In particular, k_{rs} is given by a polynomial of degree five or less or by interpolation in a table and is evaluated at the average temperature, $\frac{1}{2}(T_r+T_s)$.

A more complicated thermal conductance is a composite of two of the simple types just discussed joined by a "contact" conductance. Here it is convenient to introduce two additional network points (suppose r' and s') at the ends of the contact resistance as is shown in the following diagram:

It should be noted here that the points r' and s' are not regarded as indexed points since they are internal to a single conductance. Since there are no lumped thermal capacitances associated with the points indexed by r' and s', the thermal flux through each conductance must be the same as the flux from r to s. Thus the relations are

$$q_{rs} = q_{rr'} = q_{r's'} = q_{s's} ,$$

$$q_{rs} = K_{rs} (T_r - T_s) ,$$

$$q_{rr'} = K_{rr'} (T_r - T_{r'}) ,$$

$$q_{r's'} = K_{r's'} (T_{r'} - T_{s'}) ,$$

$$q_{s's} = K_{s's} (T_{s'} - T_s) .$$

Together these lead to the formulation:

$$K_{rs} = \left(\frac{1}{K_{rr'}} + \frac{1}{K_{r's'}} + \frac{1}{K_{s's}}\right)^{-1} \tag{6}$$

$$T_{r'} = T_r - \frac{q_{rs}}{K_{rr'}}, \quad T_{s'} = T_s + \frac{q_{rs}}{K_{s's}} \tag{7}$$

Here the contact conductance $K_{r's'}$ has the same form as that given by (5) except that A/L is replaced by A alone. As suggested by the diagram $K_{rr'}$ is calculated from the mean of T_r and $T_{r'}$, $K_{r's'}$ from that of $T_{r'}$ and $T_{s'}$, and $K_{s's}$ from that of $T_{s'}$ and T_s.

In order to compute the value of a composite conductance K_{rs}, the values of its component conductances must be known. These in turn depend on the internal temperatures $T_{r'}$ and $T_{s'}$ as well as on T_r and T_s. However, the values of $T_{r'}$ and $T_{s'}$ depend on T_r and T_s and the values of the conductances. This poses an implicit non-linear problem for the computation of $T_{r'}$, $T_{s'}$, and the value of the composite conductance. To solve this problem with adequate accuracy at each time step the following iterative procedure is used. The values of $T_{r'}$ and $T_{s'}$ at the beginning of the last time step and the just calculated values of T_r and T_s at the end of the time step are used to compute an approximating value for the component conductances and thus for the composite conductance. From this value of the composite conductance, T_r, and T_s a value of q_{rs}, and thus new values of $T_{r'}$ and $T_{s'}$ can be computed. These new values are then used to recompute the values of the conductances. This procedure may be repeated as many times as necessary in order to obtain good values of the composite conductance and the internal temperatures.

A third type of thermal conductance that we consider arises from thermal radiation. In this case q_{rs} is given by

$$q_{rs} = \sigma A F_A F_\varepsilon (T_r^4 - T_s^4) \tag{8}$$

where,

σ = Stefan-Boltzmann constant

A = surface area of element in question

F_A = geometric exchange factor

$F_\varepsilon (T_r,T_s)$ = net emissivity factor between element in question and surroundings. F_ε is evaluated at $\frac{1}{2} (T_r,T_s)$.

In this case we may write q_{rs} in the apparently linear form

$$q_{rs} = \sigma A F_A F_\varepsilon(T_r,T_s) P(T_r,T_s) (T_r-T_s)$$

where

$$P (T_r,T_s) = T_r^3 + T_r^2 T_s + T_r T_s^2 + T_s^3 \quad .$$

In this way we may obtain K_{rs} from the formula

$$K_{rs} = \sigma A F_A F_\varepsilon (T_r,T_s) P (T_r,T_s) \quad . \qquad (9)$$

Our final type of thermal conductance arises from forced convection. Here the conductance is calculated from a heat transfer coefficient h_{rs} and is given by

$$K_{rs} = A_{rs} h_{rs} \qquad (10)$$

The heat transfer coefficient itself is calculated in a more complicated way than any of the quantities that have been dealt with up to the present. The details of this computation are described in Appendix A at the end of this report. For the present it is convenient to regard it as a function which depends on time and on the temperature T_s. In the application of a conductance arising from aerodynamic heating in the heat transfer model one end point, suppose r, of each such conductance has the applied temperature $T_r$aw. The calculation of this temperature is described in Appendix A also.

In applications in the model, the heat transfer conductances usually occur along with conductances arising from radiation heat transfer to the surrounding space. Here the radiation transfer has the special form

$$q_{rs} = \sigma A_s \varepsilon(T_s) \left((T_r^{\infty})^4 - T_s^4 \right) \tag{11}$$

With the conductance given in the corresponding form:

$$K_{rs} = A_s \sigma \varepsilon(T_s) P (T_r^{\infty}, T_s) \tag{12}$$

here T_r^{∞} is a temperature that corresponds to the effective temperature of the surrounding space and $\varepsilon(T_s)$ is given in tabular form or by a polynomial. T_r^{∞} may be defined as a function of altitude or time.

III. Specification of the Model

In this section we describe in some detail the input required by the computer program which makes it possible for the IBM 7090 computer to perform the vast amount of computation involved in the model of section II.

Other questions concerning the use of the program will probably arise. Some of these questions, such as those relating to cost (Machine Time) and the size of problem which the program is capable of handling (Machine Storage), can be answered by referring to Appendix D. Other of these questions will be best answered by establishing direct liaison with BCC.

Input consists of a simple but complete FORTRAN control program prepared by the user and a set of FORTRAN-FAP subroutines (Nos. 11.02.01-11.02.12) available in the BCC Library. The control program must be prepared on standard FORTRAN coding sheets which are obtainable from BCC.

There is a considerable amount of flexibility in the method of writing the control program. For the sake of simplicity we describe a method which requires a minimal knowledge of FORTRAN and leave it to experience to suggest modifications.

The structure of a typical control program is as follows:

```
*          CØNTRØL STATEMENTS

           DIMENSIØN STATEMENTS

C          CØMMENTS

           LISTS

           CALL SET

       1   CALL TRAJ

           CALL AMBATM

           CALL FØRCER
             .
             .
             .
           CALL FØRCER

           CALL FØR ALT
             .
             .
             .
           CALL FØR ALT

           CALL AERØ
             .
             .
             .
           CALL AERØ
```

```
      CALL RAD
        .
        .
        .
      CALL RAD

      CALL CØM CØN
        .
        .
        .
      CALL CØM CØN

      CALL CØN
        .
        .
        .
      CALL CØN

      CALL CAP
        .
        .
        .
      CALL CAP

      CALL WRITE

      CALL PLØT
        .
        .
        .
      CALL PLØT

      CALL STEP

      GØ TØ 1

      END
```

The effect of this control program is to supply the FORTRAN MONITOR and the various subroutines with the information they require and to sequence the machine once per time step through the subroutines in the order in which they are called starting with statement 1.

Reference to section II and Appendix A should clarify the physical meaning of all the parameters that occur in the various CALL statements, and reference to section V should clarify the form in which they are written on the coding sheets as well as the interplay between these parameters and the LISTS. The definitions of all parameters in the various CALL statements follows:

1. CALL SET (START, STOP, TEMPIN, I, INDEX1, T1, ---, INDEXI TI)
   START, STOP - Start computation at time START and stop computation at time STOP.
   TEMPIN,I,INDEX1,T1,---INDEXI,TI - All thermal capacitors are initialized to TEMPIN, with exception of those I thermal capacitors whose indices are listed. INDEX J is the index of the Jth exceptional thermal capacitor which is initialized to temperature TJ.

2. CALL TRAJ (FOFXM,XM,FOFXA,XA)
   NOTE: The pair FOFX,X occurs in CALL TRAJ, CALL AMBATM, CALL FØR ALT, CALL FØRCER, CALL AERØ, CALL RAD, CALL CØMCØN, CALL CØN, and CALL CAP. It is used to represent a function FOFX(X) in one of the following two ways:
   (i) to define FOFX(X) as the polynomial

$$FOFX(X) = \sum_{i=o}^{N} A_i X^i$$

   write N for X in the CALL statement and write the following in the data LISTS:

$$FOFX(1) = A_N$$
$$FOFX(2) = A_{N-1}$$
$$\vdots$$
$$FOFX(N+1) = A_o$$

(ii) to define FOFX(X) as a table, write the following in the data LISTS:

$X(1) = N$.

$X(2) = X_1$

.
.
.

$X(N+1) = X_N$

$FOFX(1) = FOFX(X_1)$

.
.
.

$FOFX(N) = FOFX(X_N)$

The values FOFX(X) are then defined by linear interpolation in CALL AMBATM and by quadratic interpolation in all other CALL statements.

FOFXM,XM - This pair represents Mach number as a function of time.

FOFXA,XA - This pair represents altitude as a function of time.

3. CALL AMBATM (FOFXTO,XTO,PO,N)

FOFXTO,XTO - This pair represents ambient temperature as a function of altitude.

PO,N - Ambient pressure is determined by writing in the data LISTS.

$PO(1) = Z_1 = 0$

$PO(2) = \alpha_1$

$PO(3) = \beta_1$

$PO(4) = \gamma_1$

$PO(5) = Z_2$

$PO(6) = \alpha_2$

$PO(7) = \beta_2$

$PO(8) = \gamma_2$

.
.
.

$PO(4N-3) = Z_N$

$PO(4N-2) = \alpha_N$

$PO(4N-1) = \beta_N$

$PO(4N) = \gamma_N$

where the relationship between the α, β, γ and Z are as per Equations A5 and A6 of Appendix A.

4. CALL FØRCER (INDEX,FOFX,X)

   INDEX - This is the index of the thermal capacitor whose temperature is to be set to T.

   FOFX,X - This pair represents T as a function of time.

5. CALL FØRALT (INDEX,FOFX,X)

   INDEX - This is the index of the thermal capacitor whose temperature is to be set to T.

   FOFX,X - This pair represents T as a function of altitude.

6. CALL AERØ (ID,I,FLONOS,FOFXLM,XLM,FOFXLP,XLP,INDEX1,GEONO1,POSNO1,---, INDEXI,GEONOI,POSNOI)

   ID - This is aerodynamic heat transfer block identification number.

   I - This is number of thermal capacitors in this block.

   FLONOS - This is list of 5 numbers which describe the flow for this block. They should be presented in the data LISTS as follows:

   FLONOS(1) = Re_{cr}
   FLONOS(2) = α
   FLONOS(3) = α_t
   FLONOS(4) = C_{α}
   FLONOS(5) = C_{α_t}

   FOFXLM,XLM - This pair represents the ratio of local Mach number to ambient Mach number as a function of ambient Mach number.

   FOFXLP,XLP - This pair represents the ratio of local pressure to ambient pressure as a function of ambient Mach number.

   INDEXJ,GEONOJ,POSNOJ - This triple represents the index number, surface area A, and characteristic length X respectively of the Jth thermal capacitor in this block.

   This CALL AERØ format must be supplied for each heat transfer block.

7. CALL RAD (INDEX1,INDEX2,GEONO,FOFX,X)

   INDEX1,INDEX2 - This pair represents the indices of the two thermal capacitors which are exchanging heat by radiation.

   GEONO - This is the exchange factor $\bar{F}_A$ multiplied by the surface area A.

   FORX,X - This pair represents emissivity, ε, as a function of temperature.

8. CALL CØMCØN (INDEX1,INDEX2,GEONO1,GEONO2,GEONO3,FOFX1,X1,FOFX2,X2, FOFX3,X3)

   INDEX1,INDEX2 - This pair represents the indices of the two thermal capacitors of this composite (contact) conductance pair.

   GEONO1,GEONO2,GEONO3 - This triple represents area to length ratio $(A/L)_1$, area A, and area to length ratio $(A/L)_2$ respectively proceeding from the thermal capacitor of INDEX1 to the thermal capacitor of INDEX2.

   FOFX1,X1,FOFX2,X2,FOFX3 - These three pairs represent respectively, the conductivities k as functions of temperature proceeding from the thermal capacitor of INDEX1 to the thermal capacitor of INDEX2.

9. CALL CØN (INDEX1,INDEX2,GEONO,FOFX,X)

   INDEX1,INDEX2 - This pair represents the indices of the thermal capacitors of this conductance pair.

   GEONO - This represents the area to length ratio A/L.

   FOFX,X - This pair represents conductivity k as a function of temperature.

10. CALL CAP (INDEX,GEONO,FOFX,X)

    INDEX - This is the index number of this thermal capacitor.

    GEONO - This is volume V.

    FOFX,X - This pair represents specific heat ρc_p as a function of temperature.

11. CALL WRITE (I,TIME1,TIME2,---,TIMEN)

    I - This is the total number of complete printouts. A complete printout consists of all aerodynamic and trajectory information and the temperatures on all thermal capacitors and contact surfaces which differ from TEMPIN. Such printouts are meant for gross checks and should be used sparingly.

    TIME1,TIME2,---,TIMEN) - The numbers 1,2,---,N represent the times at which complete printout will occur.

12. CALL PLØT (ID,DELTEM,I,INDEX1,---,INDEXI)

    ID - This is the identification number of this PLØT group.

    DELTEM - A card is punched each time the temperature of the thermal capacitor whose index is INDEX1 changes by DELTEM. The card deck resulting from a complete run may be sorted on ID (columns 1-4) and then tabulated to obtain a printout of X-Y plotted to obtain a graph of temperature (columns 9-12, 13-16,---,69-72) versus time (columns 5-8).

I - This is the number of thermal capacitors in this PLØT group.
INDEX1,---,INDEXI - These indices represent the indices of the thermal capacitors whose temperatures are to be punched.

13. CALL STEP

This is a sequence instruction for the computer to re-start, timewise, with the solution of the next set of finite difference equations.

IV. Printout Format

The sheets of printout produced in the problem run contain the history and principal results of the analysis. Here are recorded the capacitance temperatures designated for printout and the contact temperatures that appear in those composite resistances that have been designated for printout.

Whenever aerodynamic heating is included in a heat transfer model it is essential to have some record of the principal quantities which influence this mode of heat transfer. To provide this information the values of the following quantities are provided at each printout time: missile altitude z, missile Mach number M_o, ambient pressure p_o, and ambient temperature T_o. In addition to this, characteristic information is printed out for each heat transfer block (CALL AERØ statement); here the recorded quantities are the local Mach number M, local pressure p, local air temperature T, local Reynolds number, adiabatic wall temperature T_{aw}, and the heat transfer coefficient, normalized by replacing x, the characteristic length, by 1; i.e. to get h, divide the values printed out by $x^{1-\alpha}$.

The program also has a subroutine available which enables output capacitance temperatures to be plotted as functions of time on BID X-Y plotters. The complexity of a given thermal problem, operator experience, time available and computer budget money available dictate the best use that can be made of the program. To help clarify the discussion a sample problem is included in the next section.

V. EXAMPLE

Figure 1 shows a wedge type airfoil made of stainless steel #347. It is seen that the airfoil has three webs. These webs are made of stainless steel #347. They are of 0.051 inch thickness. The skin thickness of the airfoil is 0.040 inch. It is assumed in this problem that the vertex angle of the wedge is 4 degrees and that one surface is at a fixed angle of attack of 4 degrees with the airstream, while the other surface is in line with the airstream. Temperature distributions are desired on all skin surfaces and webs during a prescribed flight pattern. It is recommended that the reader uses the FORTRAN sheets at the end of this report in order to follow through this sample problem.

Geometry

Figures 2 and 3 show a convenient method of disecting the physical airfoil into a lumped parameter system of capacitors.

TRAJECTORY FUNCTIONS

To define the trajectory, the altitude and free stream Mach number are prescribed as functions of time under the TRAJECTORY FUNCTIONS heading, page 2. They appear as functions F1 and F2.

AMBIENT ATMOSPHERE FUNCTIONS

It is seen in Appendix A, Eq. (A3), that in order to evaluate the expression h, the local flow conditions (pressures and velocities that exist immediately outside of the boundary layer), must be determined. These, in turn depend on the flight trajectory of the airfoil and on the distribution of ambient pressure and temperature with altitude. These last quantities are referred to under the AMBIENT ATMØSPHERE FUNCTIØNS heading. Function F3 describes the variation of ambient temperature with altitude. The function F4 describes constants to be used in the computation of ambient pressure by functions of the form given by Eq. (A5) of Appendix A. The order of listing these constants is as follows.

1st, list lowest altitude of the trajectory
2nd, list α
3rd, list β
4th, list γ

} which satisfy Eq. (A5) for ambient pressure in the lowest to intermediate altitude range.

5th, list an intermediate altitude of the trajectory

6th, list α
7th, list β
8th, list γ
} which satisfy Eq. (A5) for ambient pressure in the intermediate to high altitude range.

9th, list a high altitude of the trajectory

10th, list α
11th, list β
12th, list γ
} which satisfy Eq. (A5) for ambient pressure above this high altitude

FLOW NUMBERS

As may be seen in Eq. (A3), in order to evaluate the coefficient of convective heat transfer h, the constant C_α and the exponent α have to be specified. The constant C_α is dependent on the type of flow existing on the surface of investigation, i.e., flat plate, duct or cone, and on the transition Reynolds number which indicates when the flow changes from laminar to turbulent. The exponent is solely dependent on the transition Reynolds number. Consequently, under the FLØW NUMBERS heading, the transition Reynolds number is specified. It is followed in order, by the value of α for Reynolds number less than or equal to the critical value, the value of α for Reynolds number greater than the critical value, the value C_α for Reynolds number less or equal to critical, and the value of C_α for Reynolds number greater than critical. In our sample problem we consider flat plate theory in which we assume $R_e = 1.5 \times 10^6$, $\alpha = 0.5$ laminar, $\alpha = 0.8$ turbulent, $C_\alpha = 0.332$ laminar, and $C_\alpha = 0.0265$ turbulent. The above constants and exponents appear as functions F5.

LØCAL FLØW

The local flow conditions are obtained by the presentation of the ratio of local pressure, p, to the free stream pressure, p_o, and ratio of local Mach number, M, to the free stream Mach number, M_o, as functions of free stream Mach number, M_o. In the case of elements 1, 2, 3, 4, 5 of the airfoil it is assumed that the local Mach number and the local pressure are the same as the respective free stream Mach number and free stream pressure. Therefore, the ratio of unity appears as function F6. For the other capacitances the ratios are presented in the form of polynomial functions F7 and F8. The form M (or p) = f $(M_o) = a_n x^n + a_{n-1} x^{n-1} \ldots a_2 x^2 + a_1 x + a_o$ is

used in which the coefficients $a_o, a_1 \cdots a_n$ are such that the continuous function $f(M_o)$, closely circumscribes the discrete points M/M_o (and p/p_o) for which experimental data is available. The ratios M/M_o and p/p_o can be listed in tabular form as functions of M_o if sufficient data points are available. In that case, a quadratic interpolation is used between data points.

EMISSIVITIES

Under this heading, the effective emissivity of the exterior skin of the airfoil is listed as F9. In the sample problem it is assumed that the emissivity remains at the constant value of 0.8 at all skin temperatures.

THERMAL CØNDUCTIVITY and THERMAL HEAT CAPACITANCE

As was indicated in Section II, the thermal properties of each capacitance and conductance must be specified as functions of temperature. For the properties of this sample problem, the thermal conductivity k, and the thermal heat capacitance ρC_p are listed as polynomial functions of temperature of 1st and 3rd degree, respectively. They appear as functions F10 and F12, respectively, on page 6 of the FORTRAN sheets. These polynomials were selected by examining the proximity of the functions with the discrete points from which they were generated. In the past, these polynomials were generated on the IBM 650 computer. At present BBE has a library of such polynomials for a great variety of metals and non-metals.

CALL SET Statement (FØRTRAN page 7)

In order for the computations to be carried out it is necessary to specify the flight time at which to initiate the program and the flight time at which to cease such. This appears as 0 and 100, respectively in the CALL SET statement. This information is followed by the initial temperature that is assigned to each capacitance. In the present case this appears as 560. If there is radiation by external surfaces to space, as in the present case, it is necessary to list the temperature that is to be assigned to space. In the present case capacitor 1000 refers to space, and its temperature appears as 517. If the temperature of space varies with altitude (or flight time), it is necessary to state the initial temperature of space in the CALL SET statement and in addition, it is necessary to list

the variation of the space temperature either with altitude or with time. The former would be called out by the CALL FØRALT subroutine, while the latter by the CALL FØRCER subroutine. See parts 4 and 5 of Section III.

CALL TRAJ Statement

The CALL SET statement is followed by the CALL TRAJ statement. The latter calls out the Mach No. F1 vs. time F1T, and altitude F2 vs. time F2T functions.

CALL AMBATM Statement

The CALL AMBATM statement follows the CALL TRAJ statement. It calls out the ambient temperature F3 vs. altitude F3Z functions and the polynomial function F4 which describes the pressure as a function of altitude. The degree of the polynomial is listed after the polynomial function F4 in the CALL AMBATM statement. This is necessary in order for the computer subroutine to introduce all of the polynomial coefficients into the program run.

CALL AERØ Statement

The CALL AERØ statements specify the computations for the conductances and adiabatic wall temperatures which arise from aerodynamic heating. Each statement describes a heat transfer block, consisting of separate conductances driven by the same adiabatic wall temperature, computed from the same local flow conditions and reference temperature T', radiating to the same space temperature, but having different areas, reference lengths, and emissivities. Each block is given a block number. Blocks are numbered in consecutive order. Adjacent to the block number in the CALL AERØ statement is a number which states how many conductances are described by the block. Next to this number the function F5 from the FLØW NUMBER heading appears. This is followed by the pertinent LØCAL FLØW functions F6 or F8 and F7. It is important to note that the function describing the local Mach No. has to be listed first, and is always followed by that function which describes the local pressure. Again, these functions are followed by the degree of the polynomial. Next, the most representative element - element 6, in the sense of thickness and material properties is listed. This is followed by the

aerodynamic surface area - 0.0416 ft^2, and in turn by the characteristic length - 0.1408 ft. Then elements 1, 2, 3 etc. are defined until all elements of the heat transfer block are described.

The temperature of that capacitance which appears first in the CALL AERØ statement, is used as the Tw in Equation A2 to evaluate T' for the entire block. Otherwise, the order in which the capacitances are listed within the block is of no importance.

CALL RAD

A sample page of the CALL RAD statement is shown on FØRTRAN Sheet 8. Here, the radiation interchange between exterior skin surfaces and space is considered. First is listed the element, then the identification number for space, then the element surface area. In turn, the emissivity function F9 is listed, and in turn, is followed by the degree of this polynomial which is zero.

CALL CØMCØN

A sample page of the CALL CØMCØN Statement is shown on FØRTRAN Sheet 9. In the region where heat conduction from the surface elements to the webs is considered, the composite conductance technique described in Section II is applied. The identifying elemental numbers are listed as 1, 2, . Then, the A/L ratios are listed in the following order: first, that A/L ratio which pertains to element 1, then a large number 1 x 10^9 to make the contact resistance 0, then the A/L ratio which pertains to element 2. In turn, the polynomial function F10 which describes the thermal conductivity of the material of element 1 and its degree are listed. This is followed by the number 1 which is a multiplier for the large number previously mentioned. Thus, the product is a large number and this makes the contact resistance 0. Lastly, the polynomial function describing the thermal conductivity of element 2, and its degree are, respectively, listed.

CALL CØN

An illustrative sheet of the CALL CØN statements is shown on FØRTRAN Sheet 10. The order of listing these statements should now be apparent. The two identifying capacitance numbers are listed, and are

followed by the A/L term, and in turn, by the function describing the thermal conductivity, and the degree of the polynomial.

CALL CAP

An illustrative sheet of the CALL CAP statements is shown FØRTRAN Sheet 11. First is listed the capacitance number, then the volume in cu.ft., then the heat capacitance ρC_p (in the units of BTU/ft^3-°R) F12 polynomial function, and finally the degree of the polynomial.

CALL WRITE

Here are written the flight times at which the temperatures of the capacitors is to be printed out. First, is listed the total number of time steps 33 for which printeout is requested, then the times at which print-out is desired are stated, i.e., 0., 1.5, etc.

CALL PLØT

A subroutine exists which enables output temperatures to be punched on cards which, in turn, may be used to obtain X-Y plots. See part 12, Section III. This procedure is not described here as it has been found more convenient to plot the temperatures by hand.

CALL STEP, GØ to 1, END

This is the information necessary for the computer to continue timewise with the solution of the next set of finite difference equations, and continues until the expiration of the flight time.

Results

Figure 4 shows a typical print-out page of this sample problem with the proper identifications. Though these temperatures appear in degrees Rankine, the program has recently been modified so that temperatures now are printed out in degrees Fahrenheit. The pressure is expressed in pounds per square foot, and the altitude is expressed in feet. The

normalized heat transfer coefficient h' when divided by x^{1-a} is expressed in BTU/ft^2-sec-°R units. (This is the convective heat transfer coefficient h.)

The results of this problem are presented in the form of plots of temperature as a function of distance from the leading edge at various instants of time for both sides of the wedge. In addition, a plot of temperature vs. time is presented across one of the webs. These plots are shown in Figures 5, 6 and 7. It may be seen that temperature drops occur for the surface elements close to the web. This is due to the heat sink effect of the webs.

It is hoped that the above sample problem has sufficed in introducing the reader to the BBE-BCC heat transfer program. A more comprehensive understanding will come about in the actual usage of the program.

APPENDIX A

In the main body of this memorandum we have referred to a calculated heat transfer coefficient and accompanying driving temperature. This appendix sets forth the manner in which these quantities are calculated.

The heat transfer coefficients that are used are based on a formula of the form

$$h = C_\alpha R_e^\alpha P_r^\beta \frac{k}{x} , \tag{A1}$$

in which C_α is a constant, R_e is a Reynolds number, P_r is the Prandtl number for air, k is the thermal conductivity of air, and x is a length. In our applications we will compute these quantities from "local flow" pressures and velocities, i.e., those which exist immediately outside the boundary layer, and from an effective temperature T' given by the formula

$$T' = \mu_\alpha T_w + \nu_\alpha (1 + \theta_\alpha M^2) \quad T , \tag{A2}$$

where μ_α, ν_α, and θ_α are constants, M and T are local Mach number and temperature respectively, and T_w is the temperature of the surface to which aerodynamic heat transfer takes place.

By using the perfect gas law and associated specific heat relationships

$$\frac{p}{\rho} = gRT$$

$$C_p = C_v + \frac{gR}{J}$$

$$C_p\left(1 - \frac{1}{\gamma}\right) = \frac{gR}{J} , \quad C_p = \frac{\gamma}{\gamma-1} \quad \frac{gR}{J} ,$$

the definition of Reynolds number and Prandtl number

$$R_e = \frac{\rho v x}{\mu}$$

$$P_r = \frac{\mu C_p}{k} ,$$

the Sutherland expression for the viscosity of air as a function of temperature.

$$\mu = \mu_o \left(\frac{T^{3/2}}{T + T_c} \right),$$

and the expression (A2), the values of h can be obtained from expression (A1) as follows,

$$R_e = \frac{\rho v x}{\mu} = \frac{p v x}{gRT' \mu}$$

and

$$k = \frac{\mu C_p}{P_r} = \frac{\mu}{P_r} \frac{\gamma}{\gamma - 1} \frac{gR}{J} .$$

Therefore,

$$h = C_\alpha (pv)^\alpha x^{\alpha-1} (gR)^{-\alpha} (T')^{-\alpha} \mu^{1-\alpha} \left[P_r(T') \right]^{\beta-1} \frac{gR}{J} \frac{\gamma(T')}{\gamma(T') - 1}$$

By collecting terms,

$$h = \frac{C_\alpha}{J} (gR)^{1-\alpha} (pv)^\alpha x^{\alpha-1} \mu^{1-\alpha} (T')^{-\alpha} \left[P_r(T') \right]^{\beta-1} \frac{\gamma(T')}{\gamma(T') - 1}$$

Now, by introducing the Sutherland expression the expression for h becomes,

$$h = C_\alpha \frac{\left(\mu_o gR\right)^{1-\alpha}}{J} (pV)^\alpha x^{\alpha-1} \left\{ \left[\frac{(T')^{3/2}}{T' + T_c} \right]^{1-\alpha} (T')^{-\alpha} \left[P_r(T') \right]^{\beta-1} \frac{\gamma(T')}{\gamma(T') - 1} \right\} \qquad \text{(A3)}$$

Here C_α, J, μ_o, g and R are constants, p is the local pressure, v is the local velocity, x is a length, T' is given by (A2), T_c is a constant temperature, and γ is the ratio of specific heats for air. In the expression (A3) it may be observed that the entire quantity within the braces is a function of T' alone and its value is completely determined

when the variations of P_r and γ as functions of temperature are prescribed. Here we will calculate γ and P_r from approximations by fifth degree polynomials over the range of interest.

In order to evaluate the expression for h, the local flow conditions must be determined. These in turn depend on the trajectory of the missile flight and on the distribution of ambient pressure and temperature with altitude.

We will describe the variation of ambient temperature, T_o, with altitude, z, as a continuous piecewise linear function of altitude, i.e.,

$$T_o = \delta_i + k_i z, \quad z_{i-1} \leq z \leq z_i, \quad (i=1,2,\cdots,5), \tag{A4}$$

where δ_i and k_i are constants. Distributions of ambient pressure, P_o, hydrostatically compatible with linear temperature distributions are of one of two forms, either

$$p_o = \alpha_i \left[1 + \beta_i (z - z'_{i-1})\right]^{\gamma_i}, \quad z'_{i-1} \leq z \leq z'_i, \tag{A5}$$

or

$$P_o = \alpha_i \exp\left[\beta_i (z - z'_{i-1})\right], \quad z'_{i-1} \leq z \leq z'_i \ . \tag{A6}$$

The first is compatible with a non-constant linear temperature distribution from z'_{i-1} to z'_i, while the second corresponds to an isothermal temperature distribution over the same interval.

To define a trajectory with sufficient accuracy for most heat transfer purposes the altitude z and free stream Mach number, M_o, must be prescribed as functions of time. These are given as continuous piecewise linear functions of time. Such a function is completely defined when the coordinates are given for its initial point, for its final point, and for all "corners" at which the slope of the function changes.

The local flow conditions can then be obtained provided that the ratio of the local Mach number, M, to the free stream Mach number, M_o and the ratio of the local pressure p, to the free stream pressure, p_o, are prescribed. Here we will list these in tabular form as functions of free stream Mach number or as polynomial functions of M_o.

The remainder of the local flow conditions, i.e., the local temperature, enthalpy, and velocity are to be computed from the relations

$$H(T) + \frac{R}{2J} T \gamma(T) M^2 = H(T_o) + \frac{R}{2J} T_o \gamma(T_o) M_o^2 \qquad (A7)$$

and

$$v = [g RT \gamma(T) M^2]^{\frac{1}{2}}. \qquad (A8)$$

Here H is the enthalpy and R the "gas constant" for a pound of air and J is the conversion factor from energy in mechanical units to energy in thermal units. We will approximate H(T) over the range of interest by a polynomial of fifth degree.

Returning to the expressions for the heat transfer coefficient and T', it will be noticed that several constants have been given the subscript α, the exponent in (A1). The purpose of this notation is to indicate that during the course of the computations α may be changed, and along with it the constants indexed by it. In particular we will change α when the local Reynolds number exceeds a specified critical value associated with transition from laminar to turbulent flow. The local Reynolds number is calculated from the formulas

$$R_e = \frac{\rho v x}{\mu}, \qquad (A9)$$

$$\rho = \frac{p}{R g T} \qquad (A10)$$

$$\mu = \mu_o \frac{T^{3/2}}{T + T_c}, \qquad (A11)$$

where p, v, and T are local flow quantities.

The temperature which acts as the source or driving temperature for aerodynamic heat transfer, i.e., the "adiabatic wall" temperature, is computed on an enthalpy basis as follows:

$$H_{aw} = H(T) + r_\alpha \frac{R}{2J} T \gamma(T) M^2 \qquad \text{(A12)}$$

$$T_{aw} = T(H_{aw})$$

in which r_α is a recovery factor and T(H) is the function which gives air temperature as a function of enthalpy. This last function can be approximated over the range of interest by a polynomial of degree five.

APPENDIX B

Stability

While there is available no exact treatment of stability for the type of equations that we consider in our heat transfer model, it is reasonable to expect (and experimental results bear this out) that the same criterion that is sufficient for stability in those cases which can be treated exactly will be suitable here. Briefly, this criterion is that the inequality,

$$\Delta t \leq \frac{C_j}{\sum_i K_{ij}} ; \qquad \text{(B1)}$$

hold for all points j which have capacitance associated with them. Here, of course, the sum is taken over all indices i which are connected to j by non-zero capacitances.

This inequality may be given the following physical interpretation: If the values of the conductances and capacitances in the network are calculated for any point j for the actual temperature distribution that occurs in the model at a given time, then the time step allowable at that time is such that even if the surrounding temperatures were set equal to zero the heat that would flow out of C_j in the time Δt, i.e., $\sum_i K_{ij} T_j \Delta t$, would not be sufficient to cause the temperature $T_j + \Delta T_j$ to become negative. In this form the criterion (B1) may be regarded as an expression of the second law of thermodynamics. In our case the quantities C_j and K_{ij} are functions of temperature and are thus not known before the analysis is completed. This means that in order to evaluate the stability of the problem, we must make some a priori estimates of the temperature distributions that are expected.

The machine program has been set up so that if we set START time = STOP time in the CALL SET statement a one loop computation will result and a complete printout plus two lines:

CRITICAL INDEX NØ. = IIII
DELTIM = 0. XXXXXXXXE ± XX

will be indicated. A hand computation of Δt for thermal capacitor IIII should now be made with equation (B1). If there is more than a 10% difference

between this hand calculated Δt and the DELTIM computed by the machine a non-diagnostic error in the control program exists.

An estimate of the number of time steps required to compute from START to STOP is furnished by

$$N = \frac{\text{STØP-START}}{\text{DELTIM}}$$

APPENDIX C

BBE-BCC Finite Difference Heat Transfer Equations

The Fourier differential equation which expresses the rate of flow, or flux, of heat through a given surface element in the direction of its normal is proportional to the normal derivative of the temperature; that is,

$$\varphi = \left[- k \frac{\partial T}{\partial n} \right] dA, \tag{1}$$

where dA is the area of the surface, and k is the conductivity. Then if V is a volume enclosed by a surface S, the rate of gain of heat by the volume V is

$$\varphi = \iint_S k \frac{\partial T}{\partial n} ds = \iiint_V \nabla \cdot k \nabla T dV \tag{2}$$

where the operator ∇ (read "del") is defined as

$$\nabla = \left(i \frac{\partial}{\partial X_1} + j \frac{\partial}{\partial X_2} + \ell \frac{\partial}{\partial X_3} \right)$$

and $\nabla \cdot \; k \nabla T$ is a scalar product called the divergence of $k \nabla T$.

But, if ρ is the density and C_p the specific heat of the material involved, the rate of gain of heat is also given by

$$\varphi = \iiint_V \frac{\partial (\rho C_p) T}{\partial t} \, d V , \tag{3}$$

and so

$$\iiint_V \left(\nabla \cdot k \nabla T - \frac{\partial (\rho C_p) T}{\partial t} \right) d V = 0 , \tag{4}$$

But this is true for any volume V, so the integrand itself must vanish for every point in the material; thus we arrive at the heat equation,

$$\nabla \cdot k \nabla T = \frac{\partial(\rho C_p)T}{\partial t} \tag{5}$$

or

$$\sum_{i=1}^{3} \frac{\partial}{\partial X_i} \left[k \frac{\partial T}{\partial X_i} \right] = \frac{\partial(\rho C_p)T}{\partial t} \tag{6}$$

This is a partial differential equation which governs the flow of heat by conduction in a homogeneous or non-homogeneous, isotropic or anisotropic medium which is free of sources and sinks. One method of approximating Equations (5) or (6) for conduction in one-dimension is by the following finite difference equation,

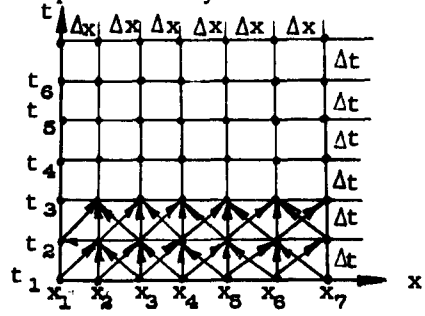
$$k_1 \left[\frac{T_{t,x+\Delta x} - T_{t,x}}{\Delta x^2} \right] + k_2 \left[\frac{T_{t,x-\Delta x} - T_{t,x}}{\Delta x^2} \right] = \left[\frac{(\rho C_p)\, T_{t+\Delta t,x} - (\rho C_p)\, T_{t,x}}{\Delta t} \right] \tag{7}$$

where $k_1 = f\,(T_{t,x+\Delta x} + T_{t,x})$

$k_2 = f\,(T_{t,x-\Delta x} + T_{t,x})$

$\rho C_p = f\,(T_{t,x})$

This may be shown pictorially as follows:



Here for a finite time interval Δt, the term $\Delta T/\Delta x$ is expressed in terms of conditions at the beginning of the time interval. Therefore, this difference equation is called a forward difference equation. (Central and backward difference equations are also possible to construct, but will not be the subject of this Appendix.)

It is seen that with the above equation the temperature at any point x, can be found at a future time $t+\Delta t$ provided only that a history of the present temperature of the point x, and neighboring points $x+\Delta x$ and $x-\Delta x$ is known. However, this equation holds only for conduction in a medium which is free of sources and sinks. If the desired internal temperature is that of a point which is adjacent to a boundary point, i.e., point x_2 next to boundary point x_1, then it is necessary to know the temperature of point x_1 at a previous time. For example, in order to evaluate the temperature of point x_2 at time t_3 by Equation (7), it is necessary to know the temperature of element x_1 at time t_2. This is found by rewriting Equation (1) in the following form,

$$\frac{\varphi}{dA} = \sum_{\substack{\text{rad,}\\ \text{conv}\\ \text{cont.res.}}} q = - k \frac{\partial T}{\partial n} \qquad (8)$$

where $\frac{\varphi}{dA}$ now refers to the heat applied at the surface of x_1 by convection and/or radiation or contact with some other medium i.e., contact resistance. Writing Equation (8) in a finite difference form leads to the following,

$$\sum_{\substack{\text{rad.}\\ \text{conv.}\\ \text{cont.res.}}} q\,(x_1,t_1,t_2) = - k\,(T_{t_1,x_1}) \frac{(T_{x_2,t_2} - T_{x_1,t_2})}{\Delta x} \qquad (9)$$

If $\sum q$ is known then the only unknown parameter in Equation (9) is T_{x_1,t_2}. Therefore, T_{x_1,t_2} is evaluated. It is then used in Equation (7) to evaluate T_{x_2} at t_3. If $\sum q$ is not known, it is always expressed in a form in which it is linearly dependent on T_{x_1,t_2}. Therefore, Equation (9) can still be solved for T_{x_1,t_2}. This linearization is somewhat demonstrated by Equations (5), (6), (9), (10), and (12) of the text. The additional necessary information

is that in the case of a non-linear equation such as Equation (8) of the text, the non-linear part K_{rs}, is always evaluated at a previous time instant, i.e., in the above discussion K_{rs} would be evaluated at t_1.

APPENDIX D

Dimensional Units

Since the quantities involved in the heat transfer calculations involve dimensional quantities it is necessary to establish the dimensional units. Throughout, the English system of units is used, i.e., foot, pound, °R, BTU, etc. Time by itself is always in seconds, but when stating thermal quantities such as conductivity we shall, for convenience, use hours.

The following table lists the units used for all quantities that must be listed on the problem sheets or appear on the printout sheets. (Since the completion of the included sample problem a modification has been made in the computer program whereby temperatures are printed out in degrees Fahrenheit rather than degrees Rankine. However, input temperatures still have to be listed in °R.)

Table of Dimensional Units

Quantity	Dimension
Initial time, t_o	second
Initial Values	°R
Time step, t	second
Atmosphere:	
α_i	lb ft^{-1}
β_i	ft^{-1}
γ_i	non-dimensional
Z_i	ft
δ_i	°R
k_i	°R ft^{-1}
Z'_i	ft
Specific Heat, ρc_p	BTU ft^{-3} $°R^{-1}$
Volume, V	ft^3
Area, A	ft^2
Area/Length, A/L	ft
Conductivity, k	BTU hr^{-1} ft^{-1} $°R^{-1}$
Stefan Boltzmann constant, σ	BTU hr^{-1} ft^{-2} $°R^{-4}$
Exponents, α	non-dimensional
Heat Transfer Constants, C_α	non-dimensional
Characteristic Length, x	ft
Heat transfer Coefficient, h	BTU sec^{-1} ft^{-2} $°R^{-1}$
Temperature, T	°R or °F
Geometric exchange factor, F_A	non-dimensional
Net emissivity factor, F_ε	non-dimensional

APPENDIX E

Machine Program

Program Restrictions

(1) Thermal capacitor indices must not exceed 1000.

(2) There must be no more than 20 CALL AERØ statements.

(3) There must be no more than 500 CALL CØMCØN statements.

(4) There must be no more than 50 CALL PLØT statements.

(5) There must be no more than 16 capacitors in any CALL PLØT statement.

Machine Timing

The machine time may generally be estimated by multiplying the machine time per time step by the number of time steps required to compute from START to STØP. (See the last part of Appendix B on how to estimate the number of time steps.) The machine time per time step may be estimated by multiplying the time required by each inner loop subroutine by the number of times it is called and summing over all inner loop subroutines.

Subroutine Name	Time
FØRCER	1 milli-second
AERØ	20 milli-seconds
RAD	1 milli-second
CØMCØN	5 milli-seconds
CON	1 milli-second
CAP	1 milli-second

Machine storage Space Requirements

(a) It will generally prove most convenient to use BCC Library Subroutines 11.02.01-11.02.12 as a package. In the somewhat unlikely event that space becomes a problem, the following information may be useful:

Number	Name	Locations
11.02.01	SET	93
11.02.02	TRAJ	40
11.02.03	AMBATM	150
11.02.04	FØRCER	40
11.02.05	AERØ	670
11.02.06	RAD	85
11.02.07	CØMCØN	100
11.02.08	CØN	65
11.02.09	CAP	40
11.02.10	WRITE	300
11.02.11	PLØT	170
11.02.12	STEP	140

The 11.02.01-11.02.12 package also includes BCC Library Subroutines DECIDE, PIF1 and PIF2. The package uses 6174 CØMCØN locations.

(b) Each CALL statement in the control program requires a certain number of locations depending on the type of CALL statement.

Type	Locations
CALL SET	5 + 2I
CALL TRAJ	5
CALL AMBATM	5
CALL FØRCER	4
CALL AERØ	8 + 3I
CALL RAD	6
CALL CØMCØN	12
CALL CØN	6
CALL CAP	5
CALL WRITE	2 + I
CALL PLØT	4 + I
CALL STEP	1

REFERENCES

1. Fox, D. W.,"Numerical Approximations in Heat Transfer Problems," BBE Internal Memorandum 3089, 2/5/60.

2. Dusinberre, G.M.,"Numerical Analysis of Heat Flow", McGraw-Hill Book Company, Inc., 1st edition, New York, 1949.

FIGURE 1. Wedge-type Wing Structure With Several Supporting Spar Webs

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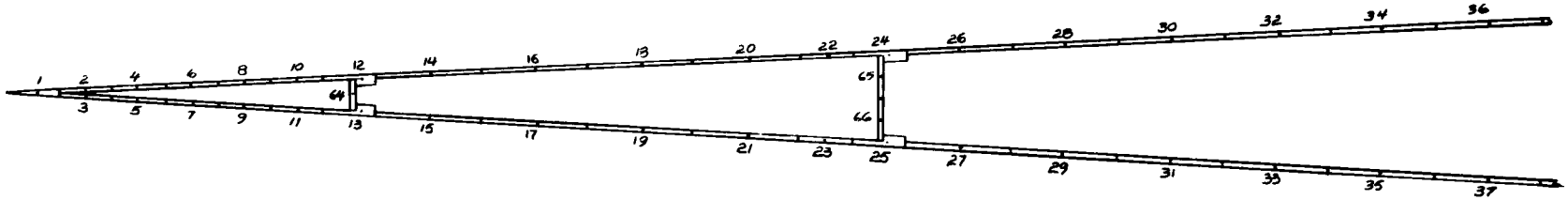
Figure 2. Tip Detail-Leading Edge Geometric Configuration of Elements Used in Heat Transfer Model



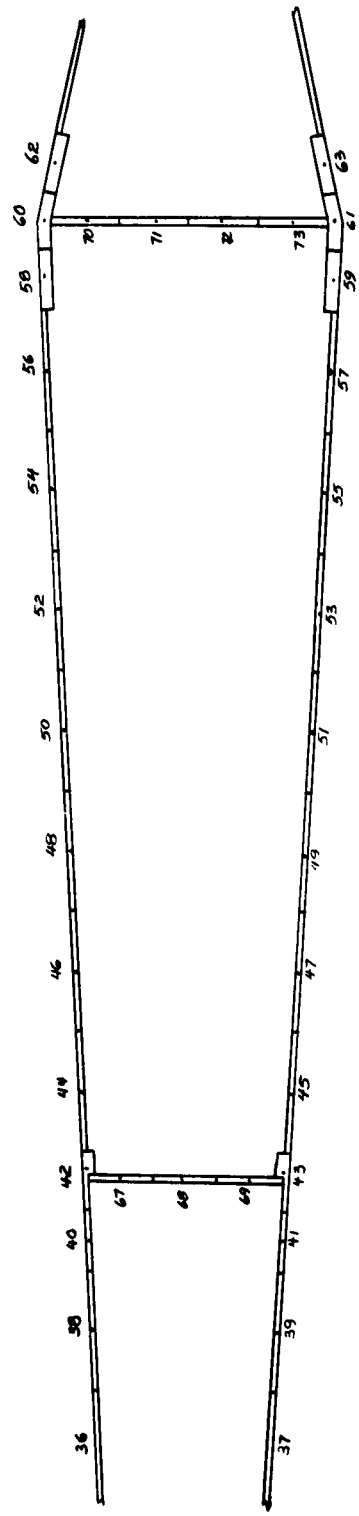


Figure 3 Geometric Configuration of Elements In Heat Transfer Model

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TIME= 0.10116858E+03 ALTITUDE= 0.1000E+06 MACH= 0.5000E+01 AMBIENT PRESSURE= 0.2335E+02 AMBIENT TEMPERATURE= 0.4254E+03

THERMAL CAPACITOR TEMPERATURES

1= 1584.	2= 1476.	3= 1524.	4= 1399.	5= 1463.	6= 1345.	7= 1414.	8= 1304.	9= 1372.	10= 1273.
11= 1332.	12= 1252.	13= 1281.	14= 1237.	15= 1290.	16= 1220.	17= 1274.	18= 1202.	19= 1255.	20= 1178.
21= 1233.	22= 1145.	23= 1198.	24= 1122.	25= 1159.	26= 1144.	27= 1194.	28= 1157.	29= 1206.	30= 1155.
31= 1203.	32= 1149.	33= 1196.	34= 1142.	35= 1189.	36= 1131.	37= 1181.	38= 1108.	39= 1164.	40= 1070.
41= 1122.	42= 1035.	43= 1073.	44= 1077.	45= 1140.	46= 1098.	47= 1169.	48= 1117.	49= 1171.	50= 1117.
51= 1170.	52= 1105.	53= 1167.	54= 1096.	55= 1156.	56= 1052.	57= 1122.	58= 970.	59= 1046.	60= 933.
61= 1006.	62= 941.	63= 1021.	64= 1262.	65= 1118.	66= 1135.	67= 996.	68= 974.	69= 1023.	70= 880.
71= 820.	72= 841.	73= 938.	1000= 517.						

CONTACT TEMPERATURES

1, 2= 1531., 1531.	1, 3= 1554., 1554.	12, 14= 1249., 1249.	13, 15= 1282., 1282.	24, 26= 1125., 1125.
25, 27= 1164., 1164.	42, 44= 1042., 1042.	43, 45= 1084., 1084.	56, 58= 984., 984.	57, 59= 1059., 1059.

ID	MACH NO.	PRESSURE	LOCAL FLOW TEMPERATURE	REYNOLDS NO.	TAW	HPRIME
1	5.00	0.2335E+02	425.	0.7140E+05	2112.	0.6640E-03
2	5.00	0.2335E+02	425.	0.9513E+05	2112.	0.6644E-03
3	5.00	0.2335E+02	425.	0.2535E+06	2112.	0.6656E-03
4	5.00	0.2335E+02	425.	0.5071E+06	2112.	0.6663E-03
5	5.00	0.2335E+02	425.	0.8448E+06	2112.	0.6667E-03
6	4.29	0.4770E+02	547.	0.9040E+05	2127.	0.9340E-03
7	4.29	0.4770E+02	547.	0.1204E+06	2127.	0.9344E-03
8	4.29	0.4770E+02	547.	0.3210E+06	2127.	0.9360E-03
9	4.29	0.4770E+02	547.	0.6420E+06	2127.	0.9369E-03
10	4.29	0.4770E+02	547.	0.1070E+07	2127.	0.9373E-03

FIGURE 4. Typical Print-out Page - Sample Problem

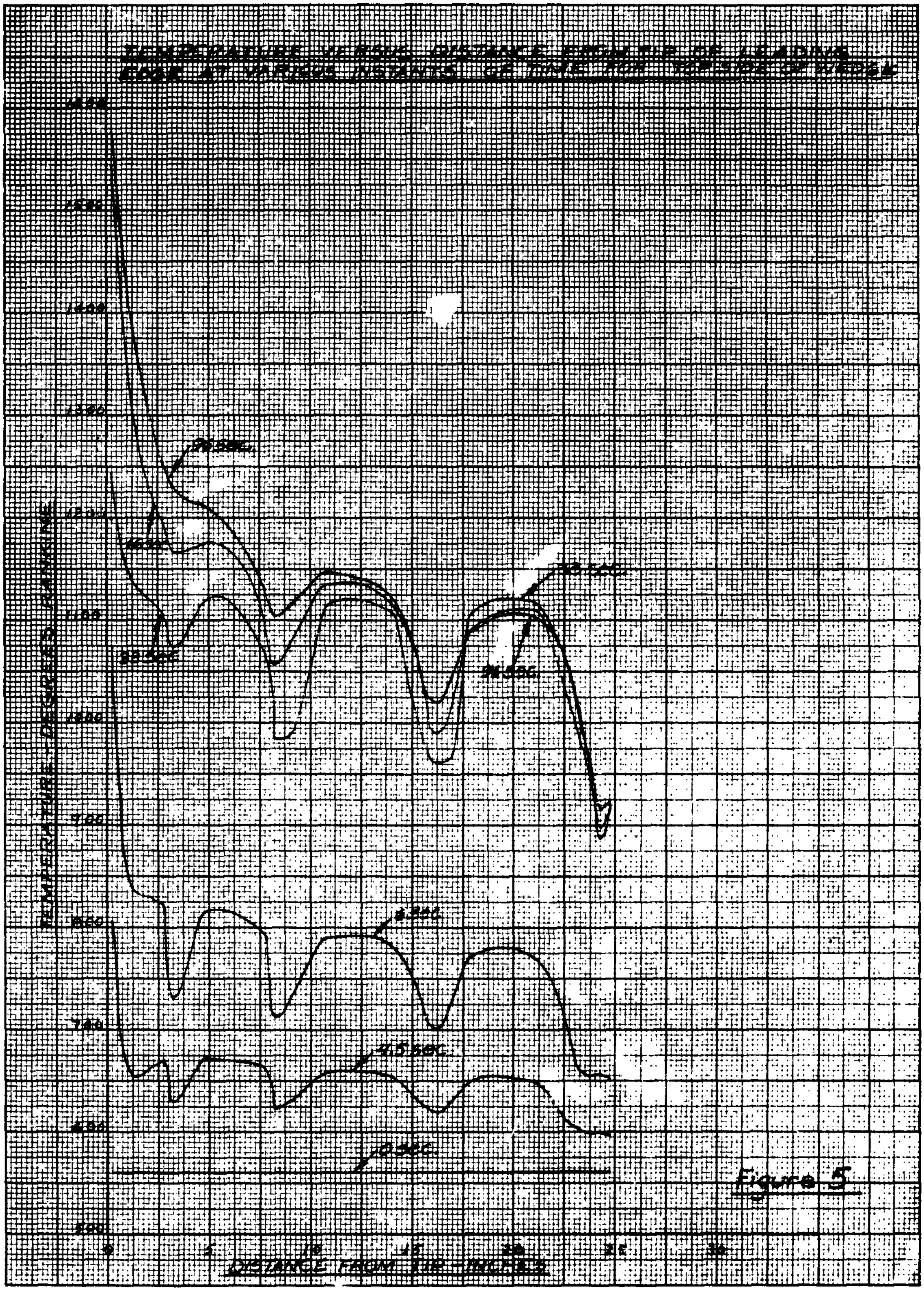
TEMPERATURE VERSUS DISTANCE FROM TIP OF LEADING EDGE AT VARIOUS INSTANTS OF TIME FOR TOP SIDE OF WEDGE
TEMPERATURE - DEGREES RANKINE
DISTANCE FROM TIP - INCHES

Figure 5

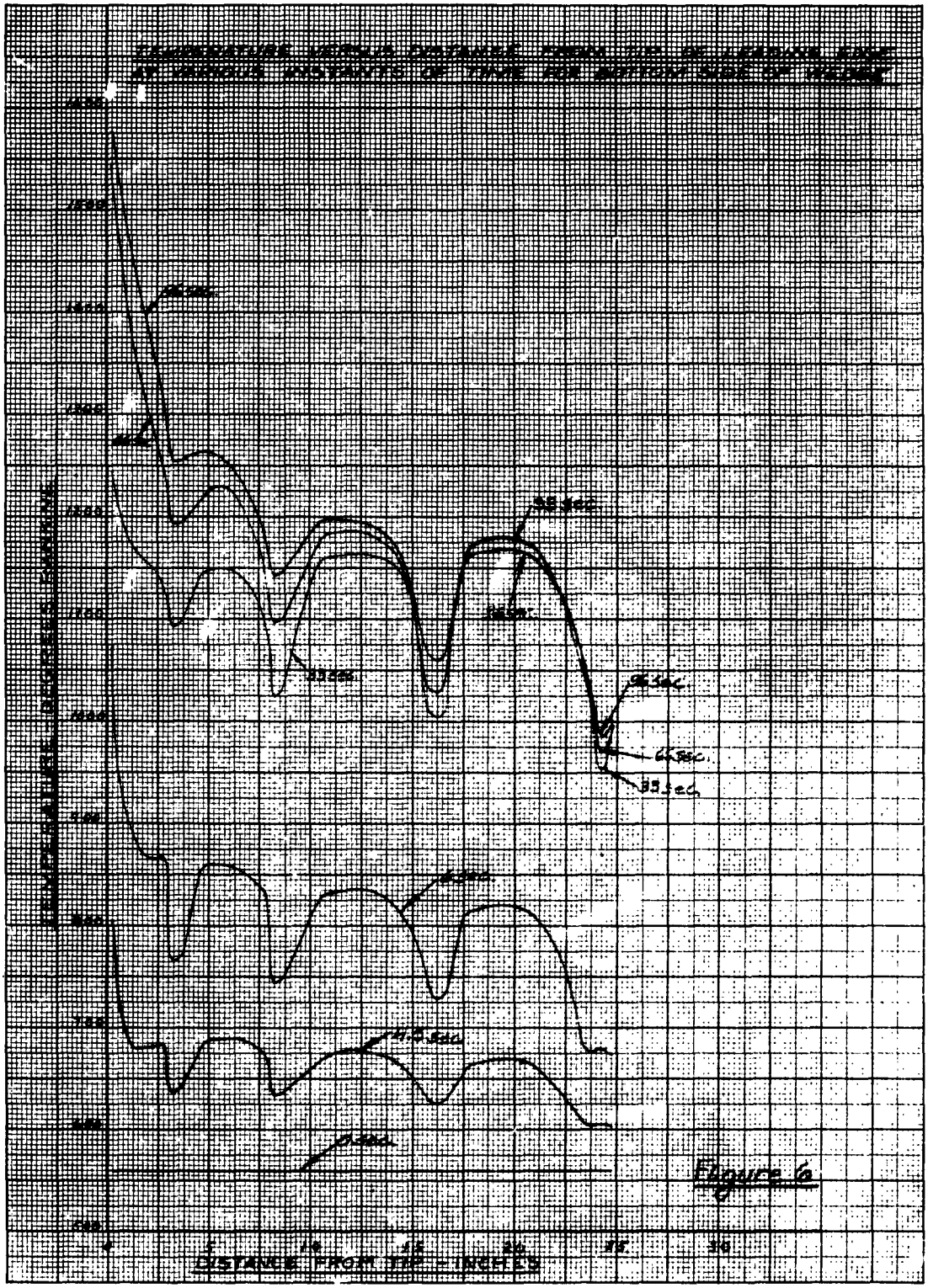
TEMPERATURE VERSUS DISTANCE FROM TIP OF CERAMIC FACE AT VARIOUS INSTANTS OF TIME FOR BOTTOM SIDE OF NOSE
DISTANCE FROM TIP - INCHES

Figure 6

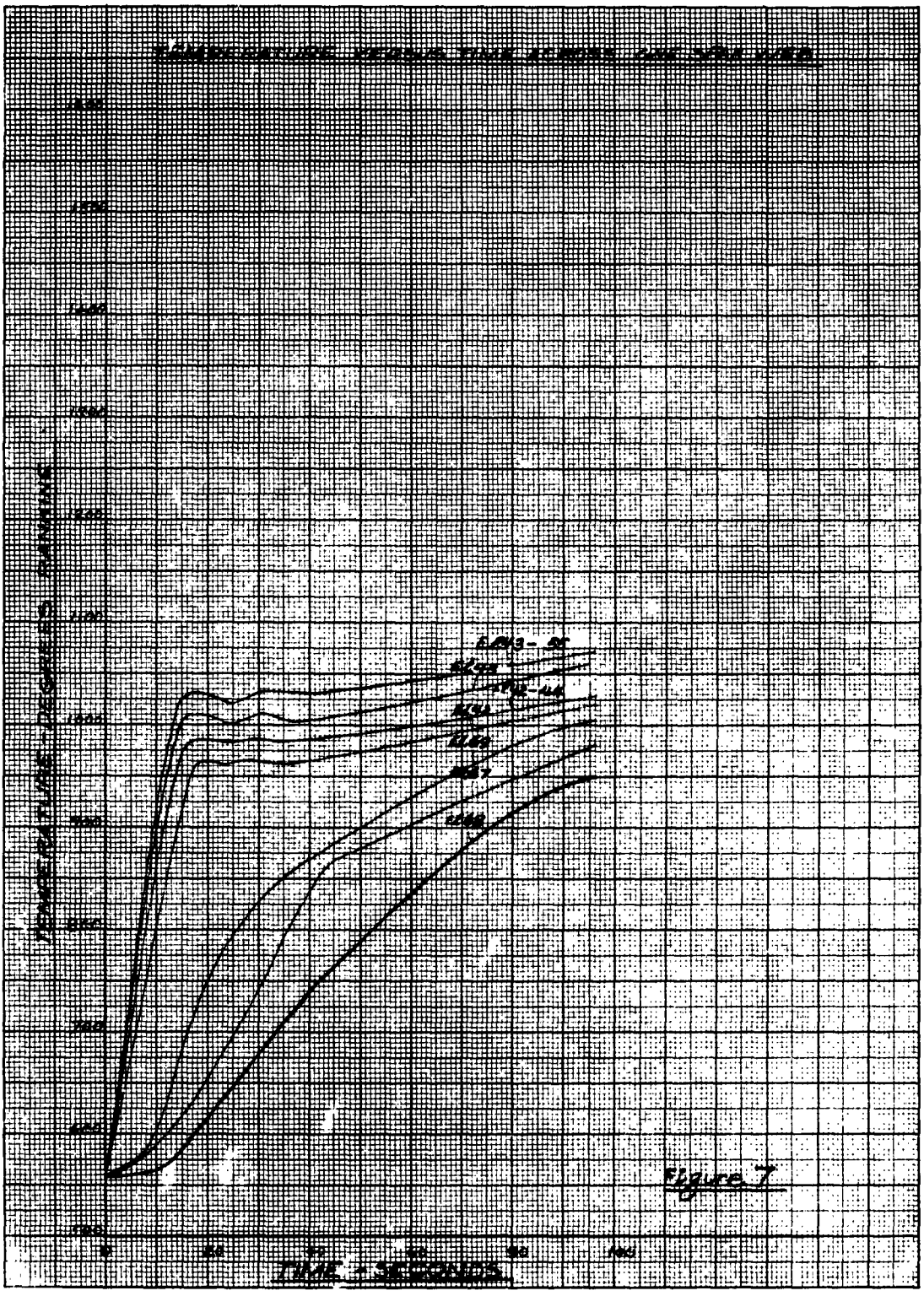

Figure 7

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PROGRAMMER Jellinek

PROBLEM NUMBER 2136

FORTRAN CODING FORM

DATE

IDENTIFICATION 73 80

C FOR COMMENT — STATEMENT NUMBER — Cont. — FORTRAN STATEMENT

Charge No — Name — Estimate Machine Time — Estimated Total lines of printout

```
*JOB          442,P,JELNEK,15,3000
*XEQ

      DIMENSION F1(12),F1T(13),F2(10),F2T(11),F3(4),F3Z(5),F4(12),F5(5),
     1F6(1),F7(5),F8(5),F9(1),F10(2),F12(4)
```

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PROGRAMMER Jellinek

PROBLEM NUMBER 2136

FORTRAN CODING FORM

DATE

IDENTIFICATION (73–80)

C FOR COMMENT

STATEMENT NUMBER (1–5)	Cont. (6)	FORTRAN STATEMENT (7–72)
		C TRAJECTORY FUNCTIONS
		C MACH VERSUS TIME
		FI(1)=0.
		FI(2)=2.56
		FI(3)=2.66
		FI(4)=3.
		FI(5)=3.03
		FI(6)=3.59
		FI(7)=3.74
		FI(8)=4.88
		FI(9)=4.97
		FI(10)=5.
		FI(11)=5.
		FI(12)=5.
		FIT(1)=12.
		FIT(2)=0.
		FIT(3)=5.
		FIT(4)=5.7
		FIT(5)=10.
		FIT(6)=11.
		FIT(7)=23.
		FIT(8)=25.
		FIT(9)=38.
		FIT(10)=39.
		FIT(11)=40.

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C FOR COMMENT

STATEMENT NUMBER	CONT.	FORTRAN STATEMENT
C		AMBIENT ATMOSPHERE FUNCTIONS
C		AMBIENT TEMPERATURE
		F3(1)=549.5
		F3(2)=347.7
		F3(3)=383.7
		F3(4)=425.4
		F3Z(1)=4.
		F3Z(2)=0.
		F3Z(3)=55000.
		F3Z(4)=70000.
		F3Z(5)=100000.
C		AMBIENT PRESSURE
		F4(1)=0.
		F4(2)=2116.22
		F4(3)=-.0000006677
		F4(4)=5.112
		E4(5)=55000.
		F4(6)=203.8
		F4(7)=.0000006902
		F4(8)=-7.819
		F4(9)=70000.
		F4(10)=94.3
		F4(11)=.0000003438

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IDENTIFICATION 73 80

C FOR COMMENT

STATEMENT NUMBER 1 2 3 4 5	Cont. 6	FORTRAN STATEMENT 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72
		F4(12)=-14.22
C		FLOW NUMBERS
		F5(1)=1.5E06
		F5(2)=.5
		F5(3)=.8
		F5(4)=.332
		F5(5)=.0265
C		LOCAL FLOW
C		PRESSURE AND MACH FOR 1,2,3,4,5
		F6(1)=1.
C		PRESSURE FOR 6,7,8,9,10
		F7(1)=-.2873291 3E-02
		F7(2)= .6848556 26E-01
		F7(3)=-.4469508 4
		F7(4)=1.2513649
		F7(5)=.1935107 89
C		MACH FOR 6,7,8,9,10
		F8(1)=.9915905 5E-02
		F8(2)= .1315649 2
		F8(3)=-.6389624 7
		F8(4)=1.3434827
		F8(5)=-.1336473 8

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C FOR COMMENT

STATEMENT NUMBER	Cont.	FORTRAN STATEMENT
C		EMISSIVITIES
C		RADIATIØN TØ FREE SPACE
		F9(1)=.8
C		THERMAL CØNDUCTIVITY
		F10(1)=.215343386E-02
		F10(2)=6.2234819
C		THERMAL HEAT CAPACITANCE
		F12(1)=.985635563E-08
		F12(2)=-.41141766E-04
		F12(3)=.67287112E-01
		F12(4)=.27539731E02
		CALL SET(0.,100.,560.,1,1000,517.)
	1	CALL TRAJ(F1,F1T,F2,F2T)
		CALL AMBATM(F3,F3Z,F4,3)
		CALLAERØ(1,4,F5,F6,0,F6,0,6,.0416,.1408,1,.0416,.0208,2,.0416,
	1	.0625,4,.0416,.0104)
		CALLAERØ(2,3,F5,F6,0,F6,0,8,.0416,.1876,10,.0416,.229,12,.0416,
	1	.271)

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IDENTIFICATION (73–80)

C FOR COMMENT

STATEMENT NUMBER (1–5) | Cont. (6) | FORTRAN STATEMENT (7–72)

```
      CALLAERØ(3,6,F5,F6,0,F6,0,18,-0833,-5,14,-0833,-333,16,-0833,-417,
     120,-0833,-583,22,-0416,-645,24,-0416,-687)

      CALLAERØ(4,9,F5,F6,0,F6,0,32,-0833,1.,26,-0833,-75,28,-0833,-833,
     130,-0833,-917,34,-0833,1-083,36,-0833,1-166,38,-0833,1-25,40,
     2-0416,1-311,42,-0416,1-355)

      CALLAERØ(5,10,F5,F6,0,F6,0,50,-0833,1-666,44,-0833,1-416,46,-0833,
     11-5,48,-0833,1-583,52,-0833,1-75,54,-0833,1-833,56,-0833,1-916,
     258,-0416,1-98,60,-0416,2-02,62,-0416)

      CALLAERØ(6,4,F5,F8,4,F7,4,7,-0416,-1408,1,-0416,-0208,3,-0416,
     1-0625,5,-0416,-104)

      CALLAERØ(7,3,F5,F8,4,F7,4,9,-0416,-1876,11,-0416,-229,13,-0416,
     1-271)

      CALLAERØ(8,6,F5,F8,4,F7,4,19,-0833,-5,15,-0833,-333,17,-0833,-417,
     121,-0833,-583,23,-0416,-645,25,-0416,-687)
      CALLAERØ(9,9,F5,F8,4,F7,4,33,-0833,1.,27,-0833,-75,29,-0833,-833,
     131,-0833,-917,35,-0833,1-083,37,-0833,1-166,39,-0833,1-25,41,
     2-0416,1-311,43,-0416,1-355)
      CALLAERØ(10,10,F5,F8,4,F7,4,51,-0833,1-666,45,-0833,1-416,47,
     1-0833,1-5,49,-0833,1-583,53,-0833,1-75,55,-0833,1-833,57,-0833,
     21-916,59,-0416,1-98,61,-0416,2-02,63,-0416,2-06)
```

PROGRAMMER Jellinek

PROBLEM NUMBER 2136

FORTRAN CODING FORM

DATE

IDENTIFICATION (73–80)

TYPICAL CALL RAD SHEET

FOR COMMENT

STATEMENT NUMBER (1–5) | Cont. (6) | FORTRAN STATEMENT (7–72)

```
      CALLRAD(48,1000,.0833,F9,0)
      CALLRAD(52,1000,.0833,F9,0)
      CALLRAD(54,1000,.0833,F9,0)
      CALLRAD(56,1000,.0833,F9,0)
      CALLRAD(58,1000,.0416,F9,0)
      CALLRAD(60,1000,.0416,F9,0)
      CALLRAD(62,1000,.0416,F9,0)
      CALLRAD(7,1000,.0416,F9,0)
      CALLRAD(1,1000,.0416,F9,0)
      CALLRAD(3,1000,.0416,F9,0)
      CALLRAD(5,1000,.0416,F9,0)
      CALLRAD(9,1000,-.0416,F9,0)
      CALLRAD(11,1000,-.0416,F9,0)
      CALLRAD(13,1000,.0416,F9,0)
      CALLRAD(19,1000,.0833,F9,0)
      CALLRAD(15,1000,.0833,F9,0)
      CALLRAD(17,1000,.0833,F9,0)
      CALLRAD(21,1000,.0833,F9,0)
      CALLRAD(23,1000,.0416,F9,0)
      CALLRAD(25,1000,.0416,F9,0)
      CALLRAD(33,1000,.0833,F9,0)
      CALLRAD(27,1000,.0833,F9,0)
      CALLRAD(29,1000,.0833,F9,0)
      CALLRAD(31,1000,.0833,F9,0)
      CALLRAD(35,1000,.0833,F9,0)
```

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PROBLEM NUMBER 2136

FORTRAN CODING FORM

DATE

IDENTIFICATION 73 80

FOR COMMENT

STATEMENT NUMBER | Cont. | FORTRAN STATEMENT

```
      CALL CØMCØN(1,2,.1586,-1E09,.1586,F10,1,F11,0,F10,1)
      CALL CØMCØN(1,3,.1586,-1E09,-1586,F10,1,F11,0,F10,1)
      CALL CØMCØN(12,14,-4086,.1E09,.0791,F10,1,F11,0,F10,1)
      CALL CØMCØN(13,15,.4086,.1E09,.0791,F10,1,F11,0,F10,1)
      CALL CØMCØN(24,26,.4086,-1E09,.0791,F10,1,F11,0,F10,1)
      CALL CØMCØN(25,27,.4086,.1E09,-0791,F10,1,F11,0,F10,1)
      CALL CØMCØN(42,44,.4086,-1E09,.0791,F10,1,F11,0,F10,1)
      CALL CØMCØN(43,45,.4086,-1E09,.0791,F10,1,F11,0,F10,1)
      CALL CØMCØN(56,58,.0791,.1E09,.4086,F10,1,F11,0,F10,1)
      CALL CØMCØN(57,59,.0791,.1E09,.4086,F10,1,F11,0,F10,1)
```

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PROBLEM NUMBER 2136

FORTRAN CODING FORM

DATE

IDENTIFICATION 73 80

TYPICAL CALL CϕN SHEET

FOR COMMENT

STATEMENT NUMBER | CONT. | FORTRAN STATEMENT

```
      CALLCϕN(31,33,.0396,F10.1)
             (32,34,.0396,F10.1)
             (33,35,.0396,F10.1)
             (34,36,.0396,F10.1)
             (35,37,.0396,F10.1)
             (36,38,.0396,F10.1)
             (37,39,.0396,F10.1)
             (38,40,.0528,F10.1)
             (39,41,.0528,F10.1)
             (40,42,.0791,F10.1)
             (41,43,.0791,F10.1)
             (42,46,.0396,F10.1)
             (45,47,.0396,F10.1)
             (46,48,.0396,F10.1)
             (47,49,.0396,F10.1)
             (48,50,.0396,F10.1)
             (49,51,.0396,F10.1)
             (50,52,.0396,F10.1)
             (51,53,.0396,F10.1)
             (52,54,.0396,F10.1)
             (53,55,.0396,F10.1)
             (54,56,.0396,F10.1)
             (55,57,.0396,F10.1)
             (58,60,.2038,F10.1)
             (59,61,.2038,F10.1)
```

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PROBLEM NUMBER 2136

FORTRAN CODING FORM

DATE _____

IDENTIFICATION _____ (73 – 80)

TYPICAL CALL CAP SHEET

FOR COMMENT | STATEMENT NUMBER (1–5) | Cont (6) | FORTRAN STATEMENT (7–72)

```
      CALL CAP (26, .27E-03, F12, 3)
      CALL CAP (27, .27E-03, F12, 3)
      CALL CAP (28, .27E-03, F12, 3)
      CALL CAP (29, .27E-03, F12, 3)
      CALL CAP (30, .27E-03, F12, 3)
      CALL CAP (31, .27E-03, F12, 3)
      CALL CAP (32, .27E-03, F12, 3)
      CALL CAP (33, .27E-03, F12, 3)
      CALL CAP (34, .27E-03, F12, 3)
      CALL CAP (35, .27E-03, F12, 3)
      CALL CAP (36, .27E-03, F12, 3)
      CALL CAP (37, .27E-03, F12, 3)
      CALL CAP (38, .27E-03, F12, 3)
      CALL CAP (39, .27E-03, F12, 3)
      CALL CAP (40, .14E-03, F12, 3)
      CALL CAP (41, .11E-03, F12, 3)
      CALL CAP (42, .23E-03, F12, 3)
      CALL CAP (43, .23E-03, F12, 3)
      CALL CAP (44, .27E-03, F12, 3)
      CALL CAP (45, .27E-03, F12, 3)
      CALL CAP (46, .27E-03, F12, 3)
      CALL CAP (47, .27E-03, F12, 3)
      CALL CAP (48, .27E-03, F12, 3)
      CALL CAP (49, .27E-03, F12, 3)
      CALL CAP (50, .27E-03, F12, 3)
      CALL CAP (51, .27E-03, F12, 3)
```

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FORTRAN CODING FORM

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C FOR COMMENT

STATEMENT NUMBER	Cont.	FORTRAN STATEMENT
		CALL WRITE(33,0.,1.5,3.,4.5,6.,7.5,10.,12.,14.,16.,20.,22.,24.,26.,
	1	28.,30.,33.,36.,39.,42.,45.,48.,51.,54.,57.,60.,66.,72.,78.,84.,
	2	90.,96.,100.)
		CALL STEP
		GO TO 1
		END

ADDENDUM

This addendum is included in order to instruct the reader in some parts of the Applied Physics Laboratory heat transfer program which need particular care. The critical aspect of the information listed below has been made evident to the BBE heat transfer section by its experience with numerous computer analyses over the past year. The problems that may arise in a computer program run, neglecting obvious technical errors, may be broken into three categories:

1. Insufficient information to obtain adequate representation of data by interpolation. This problem usually arises as a result of the fact that with the exception of the AMBATM temperature function, all tabular functions are interpolated by quadratic equations.

   Example:

   Suppose the following altitude z is presented as a function of time t,

z_1	F2(1) = 0 .	F2V(1) = N . .	
z_2	F2(2) = 2 .	F2V(2) = 0 .	t_1
z_3	F2(3) = 10 .	F2V(3) = 1 .	t_2
	. .	. .	t_3
	. .	. .	
	. .	. .	
	. .	. .	
z_N	F2(N) = 100000.	. .	
		F2V(N+1) = 100.	t_N

   where the quadratic equation

$$z = at^2 + bt + c$$

   is used.

$$\text{At } t = t_1 = 0, \quad z_1 = 0 \quad \text{therefore } c = 0$$

$$\text{at } t = t_2 = 1, \quad z_2 = 2 \quad \text{therefore,}$$

$$2 = a(1)^2 + b(1) \quad \text{therefore,}$$

$$b = 2 - a$$

$$\text{at } t = t_3 = 2, \quad 10 = a(2)^2 + (2-a)(2)$$

$$5 = 2a + 2 - a$$

$$\text{therefore,} \quad a = 3 \text{ and } b = -1,$$

$$\text{and} \quad z = 3t^2 - t.$$

Now to find the minimum value of z in the interval t_1 to t_3, differentiate z with respect to t and set equal to zero.

$$\frac{dz}{dt} = 6t - 1 = 0;$$

$$\text{therefore,} \qquad t = \frac{1}{6} \ ,$$

$$\text{and} \qquad z = 3\left(\frac{1}{6}\right)^2 - \frac{1}{6}$$

$$= -\frac{1}{12} \ .$$

Since, no ambient temperatures and pressures are listed for negative altitudes, the computer will give faulty results. Had more altitude data been given in the time interval from t_1 to t_3 then no negative z values would occur in this time interval. Therefore, the machine run would be good. In conclusion, be generous with data at the beginning, at inflection points and at ends of functional values. The best way of checking out a heat transfer program is to allow the computer to initially make only a few seconds of flight time analysis. That is in the START and STOP of the CALL SET statement, make START 0 seconds and make STOP, say, 2 seconds. That way, if something is wrong in the input information corrections can be made without wasting much computer time.

2. Inherent limitations of convective heat transfer equations. The T' - Colburn method of evaluating the coefficient of convective heat transfer is best suited to free stream Mach numbers of 5 or less. This is the method built into the Applied Physics Laboratory program. Above Mach 5 the Van Driest method of evaluating h is more accurate. Also, the equations for h must be modified for heat transfer at stagnation points. This is because of the appearance of the characteristic length $x^{1-\alpha}$ in the denominator of the equation for h.

3. Unwise assumptions. In studies involving plastic surfaces, it must be remembered that melting and/or ablating may take place at high speeds. The APL program does not handle the changes in state and geometry that may thus ensue. Therefore, computed data which may look good on surface, may be useless if above phenomena had taken place.

Another condition that one must exercise care with is choosing adequately thin elements for the representation of aerodynamic surfaces made of fiberglass phenolics or other low conductive materials. The temperature obtained for the surface element is used as the wall temperature in the computation of T'. In turn, this is used in the computation of h. For the case of low conductive materials, when the surface element is thick, then T_w will be smaller during the ascent of a missile than if the surface element were made thin because of the larger volume. In turn, the coefficient of convective heat transfer will be larger. Thus, over a finite missile ascending period of time, more heat will be supplied to the thicker surface element. Furthermore, since the overall thickness of the structure which is receiving aerodynamic heating is a fixed dimension, the inner element layers will eventually have to absorb the excessive heat supplied to the surface element. Thus, not only will the temperature of the surface element be in error, but also the temperatures of all inner elements. If the conductivity of the material is large as in the case of metals, then the above will not nearly be as serious. In conclusion, for temperature gradient supporting materials, generous usage of break-up into layers should be made in the mathematical model.

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ACKNOWLEDGMENT

The authors would like to express their thanks to Mrs. Mary Claire Auger for checking and amending the original heat transfer computer program and to Mrs. Elizabeth D. Sengstack for typing the manuscript.

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